[core:maths] support programme

A FRESH APPROACH TO A NEW QUALIFICATION

Education Development Trust

Making Connections: Number and Algebra

Mick Blaylock

mblaylock@educationdevelopmenttrust.com

Two number questions

- Work out $\frac{6}{16} \times \frac{8}{18}$
- How many $\frac{1}{4}$ inches can be fitted into $\frac{2}{3}$ of an inch?
- How do you know you are right?

Trainee teachers' responses to

$$\frac{6}{16} \times \frac{8}{18}$$

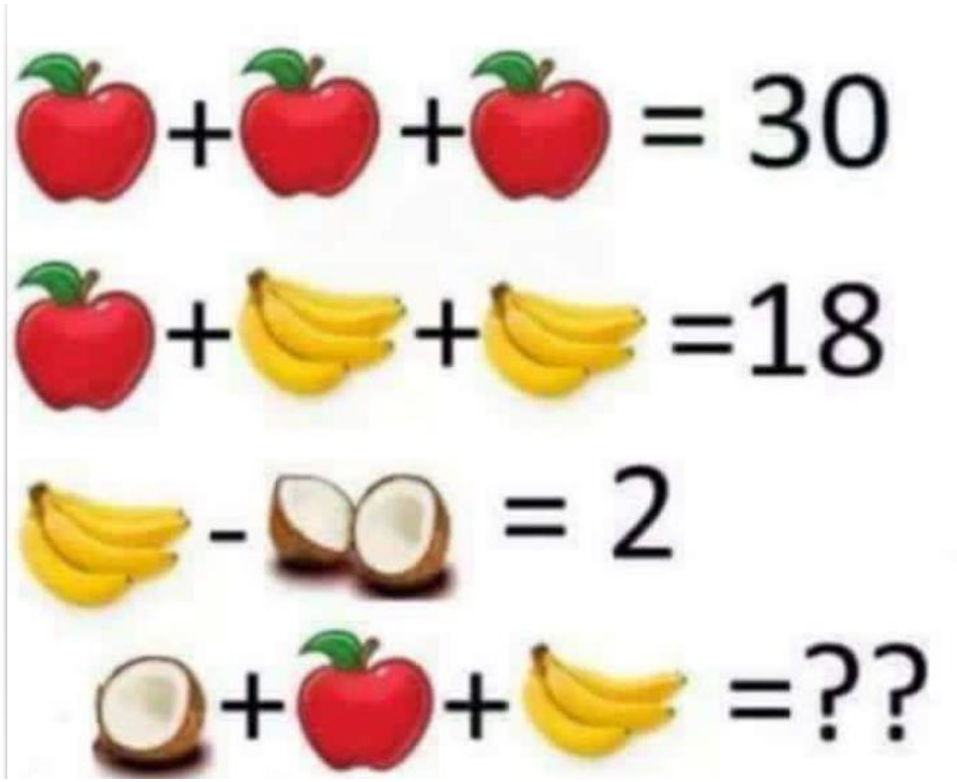
100% used a procedure to answer

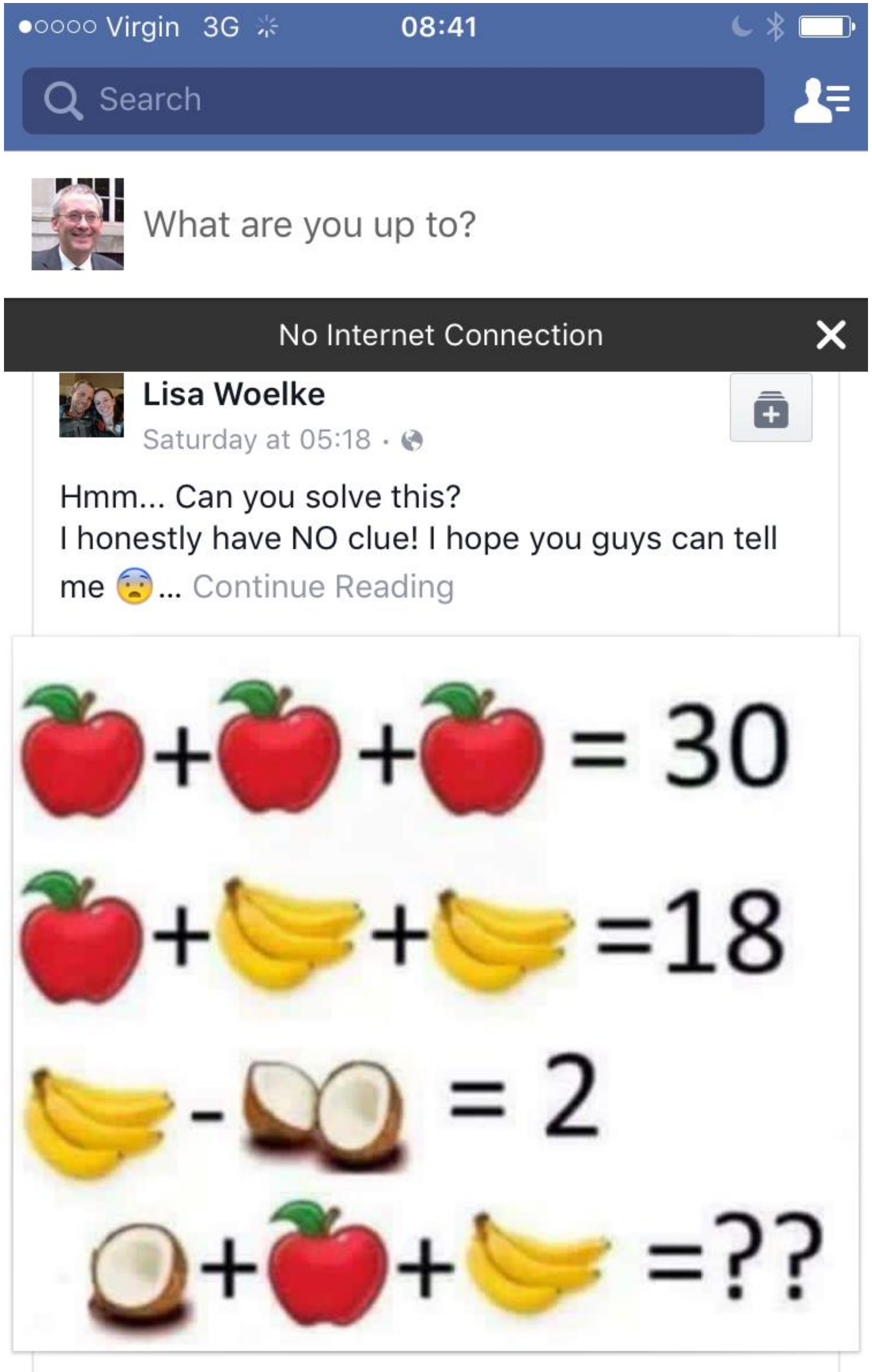
- 71% used a procedure to justify a procedure
- 0.5% used estimation
- 22.5% said they didn't know why they were right.

Trainee teachers' responses to How many $\frac{1}{4}$ inches can be fitted into $\frac{2}{3}$ of an inch?

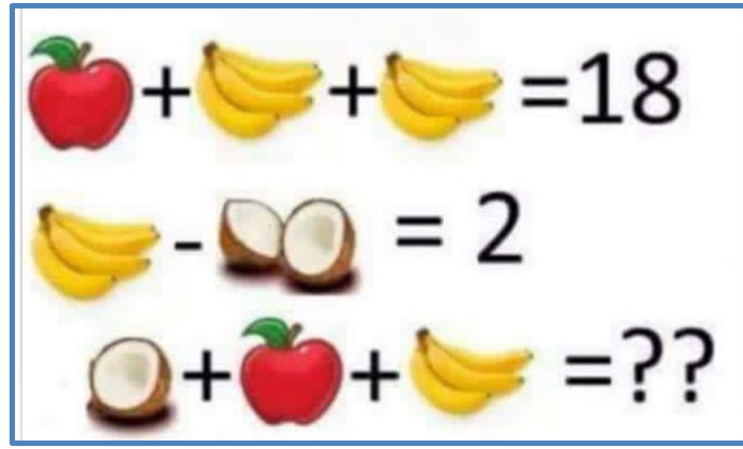
- 42% rewrote the question as $\frac{2}{3} \div \frac{1}{4}$
- 10% went for seeing how many $\frac{3}{12}$ they could fit into $\frac{8}{12}$
- 16% drew a picture

An Algebra problem?





A more interesting Algebra problem



- $x + 2y = 18$
- $y - z = 2$
- $x + y + z = 16$

- $(x + 2y) - (y - z) = 18 - 2$
- $x + y + z = 16$

Number and Algebra

- Algebraic manipulation without any meaning or purpose is a source of mystery, confusion and disaffection for adolescents.

- Relations between quantities and algebraic reasoning pervade mathematics.

Number and Algebra

Is algebra generalised arithmetic or is arithmetic an exemplification of algebra?

Number and Algebra

Algebraic reasoning involves:

- Formulating, transforming and understanding generalisations of numerical and spatial situations and relations;
- Using symbolic models to predict and explain mathematical and other situations;
- Controlling, using, understanding, and adapting spreadsheet, graphing, programming and database software.

Number and Algebra: algebra as generalised arithmetic

$$(x+2)(x+3) = x^2 + 5x + 6$$

Use numbers to check, e.g.

$x = 0$

$x = 1$

Number and Algebra: algebra as generalised arithmetic

$$(x+2)(x+3) = x^2 + 5x + 6$$

Use numbers to calculate, e.g.

$$12 \times 13 =$$

$$2.1 \times 3.1 =$$

[c:m]

Number and Algebra: generalisation

The following four statements link 3, 12 and 4:

$3 \times 4 = 12$ $4 \times 3 = 12$ $12 \div 3 = 4$ $12 \div 4 = 3$

Find four statements connecting 5, 35 and 7.

Find four statements connecting 14, 168 and 12.

This requires students applying knowledge from one situation to another and starts to open up opportunities to generalise.

Number and Algebra: generalisation

- Given ab=c write down a further three relationships between a, b and c.
- Given a+b=c write down a further seven relationships between a, b and c.

[c:m]

Number and Algebra

Challenges:

- Understanding of notation e.g. Squared and doubled

- Misuse or misremembering of techniques, e.g. Quadratic formula?

- = is equals not makes.

- Equivalence and identity, e.g. compare

$$3(x+2)=2x-7$$

and

$$3(x+2)=3x+6.$$

- Letters as labels, givens, unknowns, variables, parameters or constants!

Number and Algebra

Challenges:

- Need to distinguish between: formulae (connecting quantities), equations (x as unknown), identity (x as argument), properties and functions.
- Actions: translating, transforming, generalising, solving, simplifying, graphing, justifying, and expressing relations and structures.
- 'Knowing that 'letters stand for numbers' is not enough!

Number and Algebra

Solving equations

$$5x + 8 = 11$$

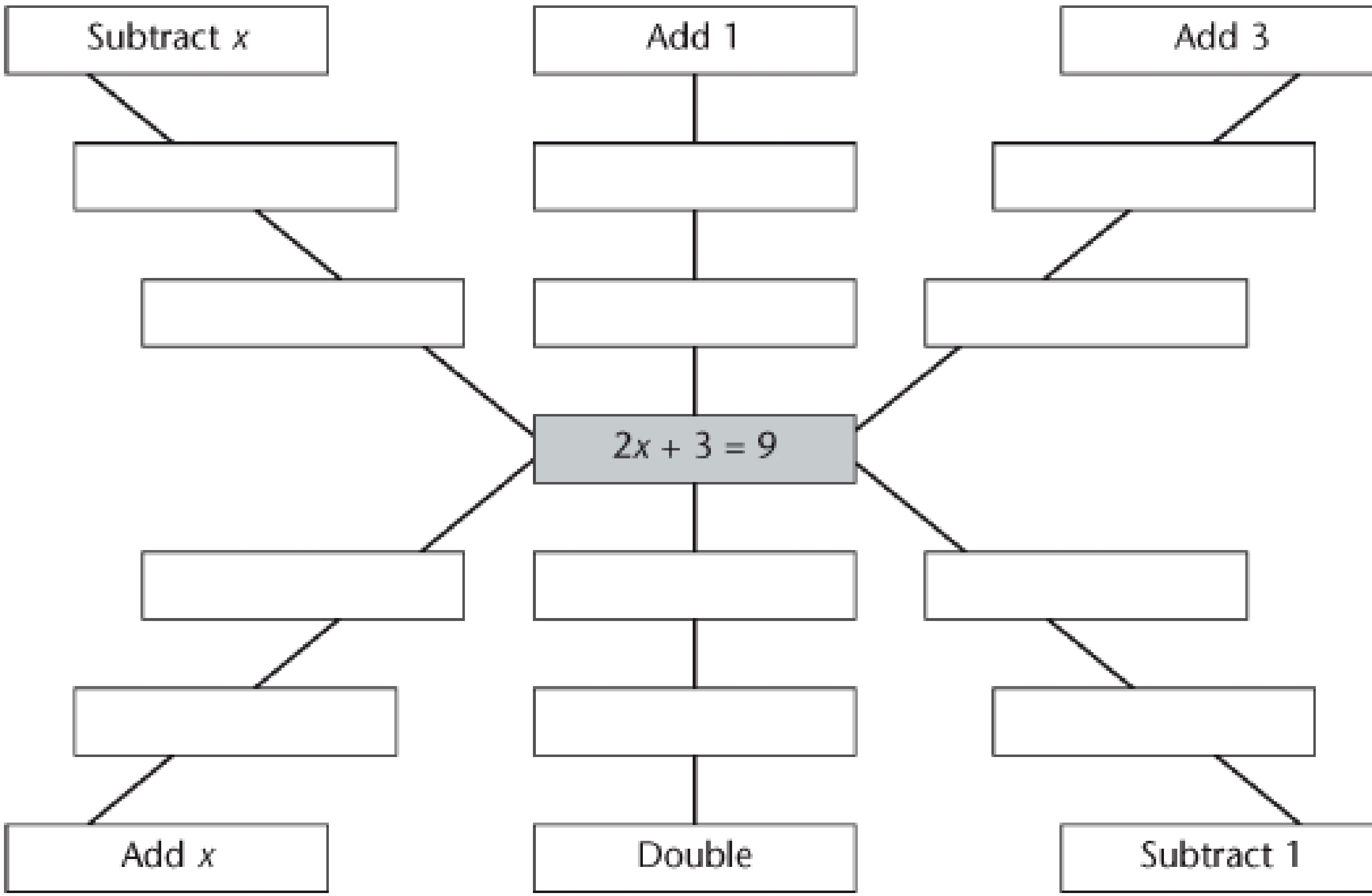
$$5x + 8 = 3 + 8$$

$$5x + 8 = x + 11$$

$$4x + x + 8 = x + 8 + 3$$

Number and Algebra

Clouding the picture: algebra 2



Number and Algebra

Learning transformation processes is only purposeful as part of problem solving, modelling, proving and conjecture activity. (Bolero 2001)

Algebra taught as a tool to express and transform situations leads students to use transformations meaningfully because they have to think about what sort of expression they need in order to solve a problem or prove a result.

Number and Algebra

For what values of the parameters p and q does the equation

$$(p + 2q)x^2 + x = 5x^2 + (3p - q)x$$

hold true for every value of x?

What is the number in the 10x10 multiplication square generated by this formula?

=$B42*I$36

Number and Algebra

Teaching approaches

- Create a need for algebra to express equivalent arithmetical relations
- Express relations among quantities
- Create a need for algebra to model phenomena
- Learn about algebraic expressions by substituting numbers in them
- Learn about expressions by 'reading' them
- Learn about expressions by modelling them diagrammatically
- Transform expressions by collecting like terms, factorising, simplifying
- Construction tasks

Number and Algebra

Key questions

- What changes, what stays the same?
- Will it always work/happen?
- Find as many ways as you can.
- Opening up questions by switching answer and question.
- Characterising all possible… (objects meeting certain constraints, questions with a given answer).

Number and Algebra

Three key steps

Estimate

Calculate

Check

Key question

‘Does it make sense?

Number and Algebra

Cognitive conflict

- Pupils do not come to lessons as ‘blank slates’ but as active learners. They have their own intuitive, perhaps incomplete, understandings of the mathematics. The role of discussion is to recognise and make explicit these understandings so that they can then be modified and refined.

- ‘Cognitive conflict’ occurs when pupils realise that there are inconsistencies between their existing beliefs and observed events. Such cognitive conflict, when resolved through reflective discussion, leads to more permanent learning than teaching methods which discourage pupils from making ‘mistakes’.

Making sense with calculus

- $\int_1^5 x^3 - 6x^2 + 5x \, dx$

[c:m]

Number and Algebra

Compound interest

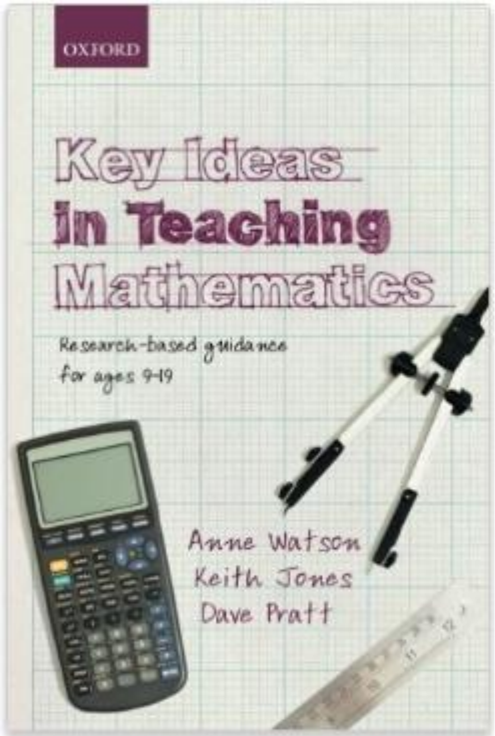
$$a(1+\frac{p}{100})^{y}$$

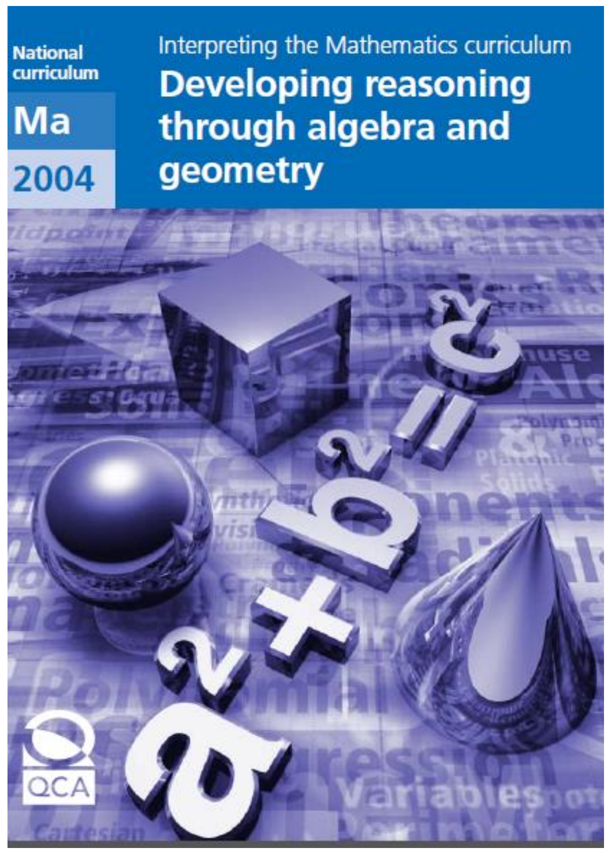
What is the effect of changing the values of *a*, *p* and *y?*

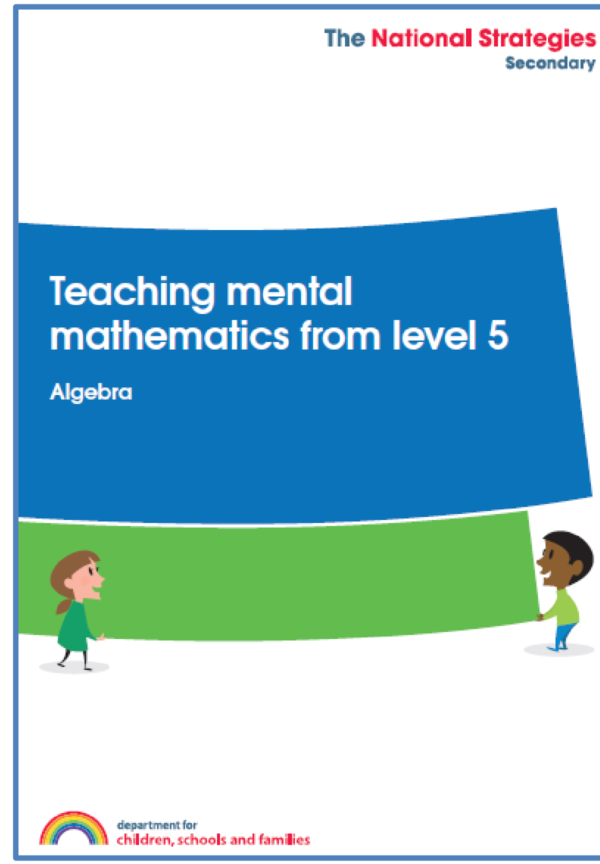
Possible questions: ‘How many years to go from £120 to £160 at 5% per annum?’ or ‘What percentage rate is needed to enable growth from £120 to £160 over four years?’.

The process of expanding the original problem and exploring the implications of doing this can make the topic more interesting for the pupils and helped to deepen their understanding of the underlying concepts.

Number and Algebra - references







c:m